LITERATURE CITED

1. Yu. P. Pshenichnikov, Detecting Fine Structure in Crystals [in Russian], Metallurgiya, Moscow (1973).

APPLICABILITY OF TIMOSHENKO-TYPE THEORIES TO LOCALIZED PLATE LOADING

P. A. Zhilin and T. P. Il'icheva UDC 539.3

1. Introduction. Many calculations have been performed on localized loading for thin bodies of plate type. The basis is provided either by Kirchhoff's theory or by the nonclassical theories of plates of Timoshenko type [1]. It is usually assumed that the two-dimensional theories of plates are not applicable directly by the point of application [1]. This is due to the essentially three-dimensional state of stress near that point.

Here we examine the state of stress and strain in a thin plate by means of the three-dimensional and two-dimensional theories. The three-dimensional theory is characterized by a singularity in the displacements of r^{-1} type, where r is the distance to the point A at which the localized force is applied. The singularity occurs only for the front surface of the plate containing the point A. It is shown here that the displacements of points in the median plane are finite. However, if the thickness 2h of the plate tends to zero, the displacements of the points in the median plane acquire a singularity of the form of $\ln r_0$, where r_0 is the distance from the point to the point A_0 representing the normal projection of point A on the median plane. The coefficient to the singularity $\ln r_0$ will be called the intensity coefficient. If we consider the displacements of the points in the three-dimensional medium averaged over the thickness of the plate instead of the displacements of the median plane, they also have a singularity of $\ln r_0$ type, but the intensity coefficient differs from that for the median plane. For $\nu = 0$ (ν is Poisson's ratio), the difference in the intensity coefficients disappears.

We now consider the two-dimensional theories. According to Kirchhoff's theory, the deflection of the plate, which is identified with the deflection of the median plane, is finite and of order $O(h^{-3})$ if one assumes that the load is of order $O(1)$ and if we take the unit of length as the least dimension of the plate in plan. The intensity coefficient in the three-dimensional theory is $O(h^{-1})$. Therefore, if h is small, the solution from Kirchhoff's theory agrees closely with the three-dimensional one in the region $|\ln r_0| \leqslant CO(h^{-1})$, where C is a bounded function of r_0, i.e., at some small distance from the point $r_0 = 0$. The solution given by a theory of Timoshenko type differs from the previous in containing a singularity in the normal displacement of $\ln r_0$ type, and there is the question of comparing the intensity coefficients obtained from the three-dimensional and two-dimensional theories.

The following treatment is based on the theory of simple shells [2-4], for which the basic relations applicable to the theory of plates are given in Sec. 3.

2. Three-Dimensional Theory. We consider a problem discussed by Galerkin [5] for a rectangular plate loaded by a distributed normal load and freely hinged at the edges, where we make certain modifications. The plate is bounded by the planes x = 0, a, y = 0, b, z = ±h. The boundary conditions take the form

$$u_2 = u_3 = \sigma_1 = 0 \text{ for } x = 0, a, \; u_1 = u_3 = \sigma_2 = 0 \text{ for } y = 0, b; \tag{2.1}$$

$$\sigma_3 = \tau_{31} = \tau_{32} = 0 \text{ for } z = -h, \; \sigma_3 = p(x, y), \; \tau_{31} = \tau_{32} = 0 \text{ for } z = h. \tag{2.2}$$

If a localized force P is applied at the point (a/2, b/2, h), the surface load takes the form

$$p(x, y) = -\frac{P}{ab}\,\delta\left(\frac{x}{a} - \frac{1}{2}\right)\delta\left(\frac{y}{b} - \frac{1}{2}\right). \tag{2.3}$$

The solution according to [5] is expressed in terms of a biharmonic function $\varphi(x, y, z)$ as follows:

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 150-155, January-February, 1984. Original article submitted August 18, 1982.

$$2Gu_1 = -\frac{\partial^2\varphi}{\partial x\partial z},\quad 2Gu_2 = -\frac{\partial^2\varphi}{\partial y\partial z},\quad 2Gu_3 = \left[2(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\varphi,\qquad \sigma_1 = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{\partial^2}{\partial x^2}\right)\varphi,\quad \tau_{12} = -\frac{\partial^3\varphi}{\partial x\partial y\partial z},$$

$$\sigma_2 = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{\partial^2}{\partial y^2}\right)\varphi,\quad \tau_{23} = \frac{\partial}{\partial y}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\varphi,\qquad \sigma_3 = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\varphi,\quad \tau_{31} = \frac{\partial}{\partial x}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\varphi,$$

where $\Delta^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplace operator and G is the shear modulus. The biharmonic function satisfying (2.1) and (2.2) takes the form

$$\varphi(x, y, z) = \sum_{m,n=1}^{\infty} [A_{mn}\operatorname{ch}\alpha_{mn}z + B_{mn}\operatorname{sh}\alpha_{mn}z + z(C_{mn}\operatorname{ch}\alpha_{mn}z + D_{mn}\operatorname{sh}\alpha_{mn}z)]\sin\lambda_m x\sin\mu_n y.$$

Here $\lambda_m = \frac{m\pi}{a},\quad \mu_n = \frac{n\pi}{b},\quad \lambda_{mn}^2 = \lambda_m^2 + \mu_n^2,$

$$A_{mn} = -\frac{\alpha_{mn}h\operatorname{sh}\alpha_{mn}h + 2\nu\operatorname{ch}\alpha_{mn}h}{\alpha_{mn}^3(\operatorname{sh}2\alpha_{mn}h - 2\alpha_{mn}h)}P_{mn},\quad C_{mn} = \frac{\operatorname{sh}\alpha_{mn}h\cdot P_{mn}}{\alpha_{mn}^2(\operatorname{sh}2\alpha_{mn}h + 2\alpha_{mn}h)},$$

$$B_{mn} = -\frac{\alpha_{mn}h\operatorname{ch}\alpha_{mn}h + 2\nu\operatorname{sh}\alpha_{mn}h}{\alpha_{mn}^3(\operatorname{sh}2\alpha_{mn}h + 2\alpha_{mn}h)}P_{mn},\quad D_{mn} = \frac{\operatorname{ch}\alpha_{mn}h\cdot P_{mn}}{\alpha_{mn}^2(\operatorname{sh}\alpha_{mn}h - 2\alpha_{mn}h)}; \tag{2.4}$$

$$p(x, y) = \sum_{m,n=1}^{\infty} P_{mn}\sin\lambda_m x\sin\mu_n y,\quad P_{mn} = \frac{4}{ab}\int_0^a\int_0^b p(x, y)\sin\lambda_m x\sin\mu_n y\,dx\,dy.$$

We now examine Galerkin's solution. For the normal deflection $u_3(x, y, z)$ we have the representation

$$2Gu_3 = \sum_{m,n=1}^{\infty}\{[2(1-2\nu)\alpha D - \alpha^2 A]\operatorname{ch}\alpha z - \alpha^2 Dz\operatorname{sh}\alpha z + [2(1-2\nu)\alpha C - \alpha^2 B]\operatorname{sh}\alpha z - \alpha^2 Cz\operatorname{ch}\alpha z\}\sin\lambda_m x\sin\mu_n y. \tag{2.5}$$

Here and subsequently, the subscripts m and n to α_{mn}, A_{mn}, B_{mn}, C_{mn}, D_{mn} are omitted for brevity.

We calculate the following quantities to compare (2.5) with data from plate theory:

$$\langle u_3\rangle = \frac{1}{2h}\int_{-h}^{h} u_3(x, y, z)\,dz,\quad w_0 = u_3(x, y, 0). \tag{2.6}$$

Also, we introduce the membrane deflection $\varphi_p(x, y)$, which is a solution to the Dirichlet problem

$$\Delta\varphi_p = -p(x, y),\ \varphi_p|_L = 0,\ \Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2 \tag{2.7}$$

in the rectangle $0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$.

The solution to (2.7) takes the form

$$\varphi_p(x, y) = \sum_{m,n=1}^{\infty}\frac{P_{mn}}{\alpha^2}\sin\lambda_m x\sin\mu_n y. \tag{2.8}$$

The function p(x, y) in (2.7) and (2.8) is the same as in (2.4). We calculate (2.6) and use (2.8) to get

$$2Gh\langle u_3\rangle = \frac{3-2\nu}{2}\varphi_p(x, y) + \sum_{m,n=1}^{\infty}\frac{2(1-\nu)hP_{mn}\sin\lambda_m x\sin\mu_n y}{\alpha(\operatorname{sh}2\alpha h - 2\alpha h)}; \tag{2.9}$$

$$2Gw_0 = \sum_{m,n=1}^{\infty}\frac{2(1-\nu)\operatorname{ch}\alpha h + \alpha h\operatorname{sh}\alpha h}{\alpha(\operatorname{sh}2\alpha h - 2\alpha h)}P_{mn}\sin\lambda_m x\sin\mu_n y. \tag{2.10}$$

If the load p(x, y) has the form of (2.3), then the membrane deflection $\varphi_p(x, y)$ has a logarithmic singularity and allows the representation [6]

$$\varphi_p(x, y) = -\frac{P}{4\pi}\ln\frac{4ab}{(2x-a)^2 + (2y-b)^2} + \theta(x, y),$$

where the harmonic function $\theta(x, y)$ is chosen such that $\varphi_p(x, y)$ becomes zero at the boundary of the rectangle $0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$. Clearly, $\theta(x, y)$ is bounded throughout the rectangle. We see that there is a substantial difference between (2.9) and (2.10): $\langle u_3 \rangle$ has a logarithmic singularity, whereas w_0 does not. The series of (2.9) and (2.10) converge uniformly throughout the region and are infinitely differentiable functions. Of course, it is assumed that $h \neq 0$.

The physical meaning of the singularity in $\langle u_3 \rangle$ is fairly clear. In fact, Galerkin's solution shows that the normal deflection has a singularity at the surface $z = h$. An infinitely thin layer near $z = h$ is separated out as a thin membrane. After averaging, this membrane solution enters into (2.9).

The series in (2.9) and (2.10) converge uniformly together with their derivatives of all orders if $h \neq 0$, while if $h \to 0$ the functions represented by (2.9) and (2.10) contain singularities, which is clearly so because the plate degenerates into a membrane for $h \to 0$, but not only to this. On calculating the asymptotes to $\langle u_3 \rangle$ and w_0, we get the representations

$$\langle u_3 \rangle = \varphi_0(x, y) + \frac{1}{2Gh} \frac{12 - 7\nu}{10} \varphi_p(x, y) + O(h); \tag{2.11}$$

$$w_0 = \varphi_0(x, y) + \frac{1}{2Gh} \frac{3(8 - 3\nu)}{20} \varphi_p(x, y) + O(h). \tag{2.12}$$

Here $\varphi_0(x, y)$ is the deflection of a freely hinged Kirchhoff plate, i.e.,

$$\Delta^2 \varphi_0(x, y) = p(x, y)/D,\ D = 2Eh^3/3(1 - \nu^2) = 4Gh^3/3(1 - \nu). \tag{2.13}$$

The function $\varphi_0(x, y)$ is $O(h^{-3})$, so if $p(x, y)$ is a smooth function we need take only the first terms in (2.11) and (2.12).

If on the other hand $p(x, y)$ takes the form of (2.3), the second terms in (2.11) and (2.12) may be the main ones and cannot be neglected near the point $x = a/2$, $y = b/2$.

Other characteristics may be examined similarly. We give expressions for some of them:

$$\langle u_1 \rangle = \frac{\nu}{4G} \frac{\partial \varphi_p}{\partial x} + O(h^2); \tag{2.14}$$

$$T_1 = \int_{-h}^{h} \sigma_1 dz = -\nu h \frac{\partial^2 \varphi_p}{\partial y^2} + O(h^3); \tag{2.15}$$

$$T_{12} = \int_{-h}^{h} \tau_{12} dz = \nu h \frac{\partial^2 \varphi_p}{\partial x \partial y} + O(h^3); \tag{2.16}$$

$$N_1 = \int_{-h}^{h} \tau_{13} dz = -D \frac{\partial}{\partial x} (\Delta \varphi_0); \tag{2.17}$$

$$M_1 = \int_{-h}^{h} \sigma_1 z dz = -D \left(\frac{\partial^2 \varphi_0}{\partial x^2} + \nu \frac{\partial^2 \varphi_0}{\partial y^2} \right) - \frac{2\nu h^2}{5} \frac{\partial^2 \varphi_p}{\partial y^2} + O(h^4); \tag{2.18}$$

$$M_{12} = \int_{-h}^{h} \tau_{12} z dz = -D(1 - \nu) \frac{\partial^2 \varphi_0}{\partial x \partial y} + \frac{2\nu h^2}{5} \frac{\partial^2 \varphi_p}{\partial x \partial y} + O(h^4). \tag{2.19}$$

Note that expression (2.17) for the shearing force is exact, whereas (2.14)-(2.16), (2.18), and (2.19) have been derived asymptotically. The formulas for $\langle u_2 \rangle$, T_2, N_2, M_2 have been derived from those for $\langle u_1 \rangle$, T_1, N_1, M_1 by means of the substitutions $\partial/\partial x \rightleftarrows \partial/\partial y$.

It follows from (2.12) that if the load takes the form of (2.3) the displacements of points in the median plane will also have a singularity of the form $\ln r_0$ for $h \to 0$. On comparing (2.11) and (2.12) we see that $\langle u_3 \rangle$ and w_0 have identical singularities for $h \to 0$, but the intensity coefficients are different. The difference in these coefficients vanishes for $\nu = 0$.

3. Plate Theory. If a thin parallelepiped is considered as a plate, then the equations of equilibrium can be written in the usual form [7]:

$$\partial T_1/\partial x + \partial T_{21}/\partial y = 0,\ \partial T_{12}/\partial x + \partial T_2/\partial y = 0; \tag{3.1}$$

$$N_1 = \partial M_1/\partial x + \partial M_{21}/\partial y,\ N_2 = \partial M_2/\partial y + \partial M_{12}/\partial x,\ \partial N_1/\partial x + \partial N_2/\partial y = -p(x, y). \tag{3.2}$$

The elastic relations in the theory of simple plates take the form [4]

$$T_1 = \frac{\nu h}{1-\nu} p(x, y) + \frac{2Eh}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2),\ T_{12} = 2Gh\omega; \tag{3.3}$$

$$M_1 = \frac{\nu h^2}{3(1-\nu)} p(x, y) + \frac{2Eh^3}{3(1-\nu^2)}(\varkappa_1 + \nu\varkappa_2),\quad M_{12} = \frac{Eh^3}{3(1+\nu)}(\tau_1 + \tau_2); \tag{3.4}$$

$$N_1 = 2Gh\Gamma_0\gamma_1. \tag{3.5}$$

The other five relations are derived from the above by the substitution 1 ⇄ 2 in the subscripts.

The following is the relationship between the strains and the displacements and rotations:

$$\varepsilon_1 = \partial u/\partial x,\ \varepsilon_2 = \partial v/\partial y,\ \omega = \partial v/\partial x + \partial u/\partial y; \tag{3.6}$$

$$\varkappa_1 = \partial\varphi_1/\partial x,\ \varkappa_2 = \partial\varphi_2/\partial y,\ \tau_1 + \tau_2 = \partial\varphi_2/\partial x + \partial\varphi_1/\partial y; \tag{3.7}$$

$$\gamma_1 = \varphi_1 + \partial w/\partial x,\ \gamma_2 = \varphi_2 + \partial w/\partial y, \tag{3.8}$$

where u, v, and w are the displacements of particles in the plate and φ_1 and φ_2 are the angles of rotation of these particles.

If we adopt Kirchhoff's hypothesis, the rigidity $Gh\Gamma_0$ in transverse shear should tend to infinity, whereas for the shearing forces N_α (α = 1, 2) to be finite it is necessary that $\gamma_\alpha \to 0$:

$$\gamma_\alpha = 0 \Rightarrow \varphi_1 = -\partial w/\partial x,\ \varphi_2 = -\partial w/\partial y. \tag{3.9}$$

We substitute (3.9) into (3.7) and (3.7) into (3.4) to get Gol'denveizer's relations [8]. However, the meanings of the displacements and rotations in this theory of simple plates differ from those in traditional versions, namely: The following relations apply:

$$u = \langle u_1 \rangle,\ v = \langle u_2 \rangle,\ w = \langle u_3 \rangle.$$

The angles of rotation φ_α in the plate are related to the displacements of particles in a three-dimensional medium by

$$\varphi_\alpha(x, y) = \frac{3}{2h^3}\int_{-h}^{h} u_\alpha(x, y, z)\, z\,dz\ (\alpha = 1, 2).$$

The boundary conditions corresponding to (2.1) in plate theory take the form

$$v = w = \varphi_2 = 0,\ T_1 = M_1 = 0 \text{ at } x = 0, a; \tag{3.10}$$

$$u = w = \varphi_1 = 0,\ T_2 = M_2 = 0 \text{ at } y = 0, b. \tag{3.11}$$

The boundary-value problem of (3.1)-(3.8), (3.10), (3.11) has an obvious solution, which is given here without derivation:

$$u = \frac{\nu}{4G}\frac{\partial\varphi_p}{\partial x},\quad \varphi_1 = \frac{\partial}{\partial x}\left(\frac{\nu\varphi_p}{4Gh} - \varphi_0\right); \tag{3.12}$$

$$w = \varphi_0(x, y) + \frac{1}{2Gh}\left(\frac{1}{\Gamma_0} - \frac{\nu}{2}\right)\varphi_p(x, y); \tag{3.13}$$

$$T_1 = -\nu h\frac{\partial^2\varphi_p}{\partial y^2},\quad T_{12} = \nu h\frac{\partial^2\varphi_p}{\partial x\partial y},\quad N_1 = -D\frac{\partial}{\partial x}(\Delta\varphi_0); \tag{3.14}$$

$$M_1 = -D\left(\frac{\partial^2\varphi_0}{\partial x^2} + \nu\frac{\partial^2\varphi_0}{\partial y^2}\right) - \frac{\nu h^2}{3}\frac{\partial^2\varphi_p}{\partial y^2}; \tag{3.15}$$

$$M_{12} = -D(1-\nu)\frac{\partial^2\varphi_0}{\partial x\partial y} + \frac{\nu h^2}{3}\frac{\partial^2\varphi_p}{\partial x\partial y}. \tag{3.16}$$

The formulas for v, φ_2, T_2, N_2, M_2 are derived from those for u, φ_1, T_1, N_1, M_1 by the substitution $\partial/\partial x \rightleftarrows \partial/\partial y$. In these expressions, the functions $\varphi_0(x, y)$ and $\varphi_p(x, y)$ have the same meaning as in Sec. 2

The solution from Kirchhoff's theory is obtained from (3.12)-(3.16) if we assume that $\varphi_p(x, y) \equiv 0$; the solution given by Reisner's theory takes the form

$$w = \varphi_0(x, y) + (3/5Gh)\varphi_p(x, y), \ u = v = 0. \tag{3.17}$$

The solution from Gol'denveizer's theory is obtained from (3.12)-(3.16) with $\Gamma_0 = \infty$.

Discussion. We compare the results of (3.12)-(3.16) from the two-dimensional theory of Timoshenko type with the consequences (2.11), (2.12), (2.15)-(2.19) of the three-dimensional theory, which shows that all quantities coincide up to $O(h^2)$ if p(x, y) is a smooth function.

Then for a smooth load we get identical expressions for the normal deflection, moments, and shearing forces from Kirchhoff's theory, from the theory of Timoshenko type, and from the three-dimensional theory for a thin plate.

If on the other hand the load has the form of (2.3), the position is radically altered. In that case, $\varphi_p(x, y)$ has a logarithmic singularity and one cannot neglect terms containing $\varphi_p(x, y)$. On comparing (2.11), (2.12), (3.13) we see that $\langle u_3 \rangle$, w_0, w have identical singularities of $\ln r_0$ type for $h \to 0$, while as regards the intensity coefficients, any coincidence is dependent on the value of the shear coefficient Γ_0. If $\Gamma_0 = 5/(6 - \nu)$, the two-dimensional theory gives an intensity coefficient the same as the solution from the three-dimensional theory $\langle u_3 \rangle$ averaged over the thickness. On the other hand, the displacement w_0 of the median plane has a different intensity coefficient. Also, for $h \neq 0$, the displacement of the median surface of (2.10) does not have a singularity.

An interesting point is that Reisner's theory gives $\Gamma_0 = 5/6$ and if $\nu = 0$ the expression (3.17) for the normal deflection coincides with the consequence from the three-dimensional theory, whereas it does not for $\nu \neq 0$. Also, there are differences in the expressions for the tangential displacements, but these play only a secondary part.

The solution from Gol'denveizer's theory contains the same singularity as the average solution from the three-dimensional theory, but these results differ in intensity coefficient.

Therefore, if we identify the solution from a two-dimensional theory of Timoshenko type with the solution from the three-dimensional theory averaged over the thickness, for $\Gamma_0 = 5/(6 - \nu)$ we get close agreement in qualitative and quantitative respects (there are identical singularities and identical intensity coefficients). If on the other hand we identify the deflection in the two-dimensional theory with the displacement of the median surface, then for $h \neq 0$ we get a qualitative difference between the results, since the displacement of the median surface does not have a singularity, while for $h \to 0$ the singularities will be of the same form but the intensity coefficients will differ.

Also, Kirchhoff's theory gives a result qualitatively different from the consequences of the three-dimensional theory, since according to Kirchhoff's theory the deflection is bounded.

Comparison of (2.17) and (3.14) for the shearing forces shows that they coincide exactly. Also, as the expressions for the shearing forces contain not the function $\varphi_0(x, y)$ itself but only its Laplacian $\Delta\varphi_0$, the series representing the shearing forces may be summed by means of elliptic functions [9].

As the plate is rectangular, in the case of free hinging at the edge we should have $\Delta\varphi_0|_L = 0$.

We solve (2.13) for $\Delta\varphi_0$ to get

$$\Delta\varphi_0(x, y) = -\frac{1}{D}\int_0^a\int_0^b K(x, y; \xi, \eta)\, p(\xi, \eta)\, d\xi d\eta, \tag{3.18}$$

where K(x, y; ξ, η) is the Green's function for the Laplace equation. This in turn can be represented [6] as

$$K(x, y; \xi, \eta) = -\frac{1}{2\pi}\ln|f(z)| = -\frac{1}{2\pi}\operatorname{Re}\ln f(z), \tag{3.19}$$

where f(z) is a function of the complex variable that maps a rectangle in the plane $z = x + iy$ into unit circle in the plane $\zeta = \xi + i\eta$.

That function is [10]

$$f(z) = \frac{\sigma(z - \zeta)\,\sigma(z + \zeta)}{\sigma(z - \bar{\zeta})\,\sigma(z + \bar{\zeta})}, \quad z = x + iy, \quad \zeta = \xi + i\eta,$$

where $\sigma(U)$ is the Weierstrass sigma function with half-periods $\omega_1 = a$, $\omega_2 = ib$.

We express the sigma function in terms of the theta function [11] and use (3.18) and (3.19) to get for the case of a plate loaded by a localized force that

$$\Delta\varphi_0(x, y) = -\frac{P}{D}K(x, y; \xi, \eta) = \frac{P}{2\pi D}\operatorname{Re}\ln\frac{\theta_1\left(\frac{z-\zeta}{2a}\right)\theta_1\left(\frac{z+\zeta}{2a}\right)}{\theta_1\left(\frac{z-\bar{\zeta}}{2a}\right)\theta_1\left(\frac{z+\bar{\zeta}}{2a}\right)},$$

where P is the intensity of the localized force and (ξ, η) is the point of application.

From (3.14) the expression for the shearing force takes the form

$$N_r = -\frac{P}{4\pi a}\operatorname{Re}\left[\frac{\theta_1'\left(\frac{z-\zeta}{2a}\right)}{\theta_1\left(\frac{z-\zeta}{2a}\right)} + \frac{\theta_1'\left(\frac{z+\zeta}{2a}\right)}{\theta_1\left(\frac{z+\zeta}{2a}\right)} - \frac{\theta_1'\left(\frac{z-\bar{\zeta}}{2a}\right)}{\theta_1\left(\frac{z-\bar{\zeta}}{2a}\right)} - \frac{\theta_1'\left(\frac{z+\bar{\zeta}}{2a}\right)}{\theta_1\left(\frac{z+\bar{\zeta}}{2a}\right)}\right]. \tag{3.20}$$

The function $\theta_1(v)$ has zeros at the points $v = m + nbi/a$, where m and n are integers. Consequently, the functions appearing in the denominators in (3.20) will have zeros at the points

$$z = \pm\zeta + 2am + 2bni,\ z = \pm\bar{\zeta} + 2am + 2bni.$$

However, the rectangle $0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$ contains only one point $z = \zeta$, the point of application of the localized force. Therefore, expression (3.20) will be everywhere bounded, apart from the point of application.

LITERATURE CITED

1. N. A. Kil'chevskii, G. A. Izdebskaya, and L. M. Kiselevskaya, Lectures on the Analytical Mechanics of Shells [in Russian], Vishcha Shkola, Kiev (1974).
2. P. A. Zhilin, "Mechanics of deformable directed surfaces," Int. J. Solids and Structures, 12, 635 (1976).
3. P. A. Zhilin, "Theory of elastic simple shells," in: Annotations for the Fifth All-Union Conference on Theoretical and Applied Mechanics [in Russian], Nauka, Alma Ata (1981).
4. P. A. Zhilin, "Basic equations in the nonclassical theory of shells," Trudy Leningr. Politekhn. Instituta, No. 386 (1982).
5. B. G. Galerkin, "Elastic rectangular and triangular freely hinged thick plates subject to bending," Dokl. Akad. Nauk SSSR, No. 10 (1931).
6. R. Courant and D. Hilbert, Methods of Mathematical Physics, Wiley (1953).
7. V. V. Novozhilov, Theory of Thin Shells [in Russian], Sudostroenie, Leningrad (1962).
8. A. L. Gol'denveizer, Theory of Elastic Thin Shells [in Russian], Nauka, Moscow (1976).
9. G. Yu. Dzhanelidze, "Determination of shearing forces in the bending of hinged thin plates," Prikl. Mat. Mekh., 10, No. 2 (1946).
10. V. I. Smirnov, Textbook of Higher Mathematics [in Russian], Vol. 3, Sec. 2, Nauka, Moscow (1974).
11. H. Bateman and A. Erdelyi, Higher Transcendental Functions, Vol. 3, McGraw-Hill.